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EXPERIMENTS ON THE SINGLE SCATTERING OF ELECTRONS

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In honor of the 70th birthday of Otto Hahn

ABSTRACT

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/88

Narrow energy bands of several 100 kev were magnetically separated from the β radiation of a powerful RaD + E preparation. These were used to investigate scattering in thin foil of varying atomic number at a mean angle of 110°. The spectrum of the scattered electrons was recorded with a magnetic half-circle spectrometer. Results: 1. Energy losses considerably in excess of those associated with bremsstrahlung were observed only when the thickness of the layer was not sufficiently small; under conditions of pure nuclear single scattering there were no anomalous losses (with certain reservations in the case of the lightest elements). 2. The dependence of the scattering cross section on the atomic number obeys the Mott formula, as corrected by Sexl and Urban, within the limits of error of theory and experiment. Greater deviations are observed with respect to the original Mott formula.

1. OBJECT OF INVESTIGATION

As the history of the problem shows, the accurate measurement of the single scattering of electrons is attended by considerable difficulties, much more so than, for example, the corresponding α-ray experiments. Accordingly, the results obtained thus far are highly conflicting. The experiments to be described relate to two problems: the velocity distribution of electrons that have undergone single scattering at atomic nuclei, and the deviation of the scattering cross section from the classical Rutherford formula.

On the basis of cloud chamber photographs, various authors have concluded that the single scattering of electrons at nuclei involves energy losses much too frequent to be attributable to radiation losses resulting from the generation of ordinary bremsstrahlung[1]. The energy spectrum of such electrons appears to contain a continuous band, extending from the primary energy in the direction of lower energies with decreasing intensity. In view of its fundamental importance, we have now investigated the problem by another method: the energy distribution of electrons scattered by thin foils was investigated with the aid of a magnetic spectrometer.

These measurements have also made possible a relative verification of the quantum mechanical theory of nuclear single scattering. Two somewhat different formulas are available: Mott's original formula[2] and one derived by Sexl[3] and Urban[4]. In view of the large scattering angle employed, an experimental comparison of the two formulas appeared possible. Accordingly, we investigated the variation with the atomic number. The apparatus was less well suited for absolute measurements of scattering cross sections.

2. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. The electron

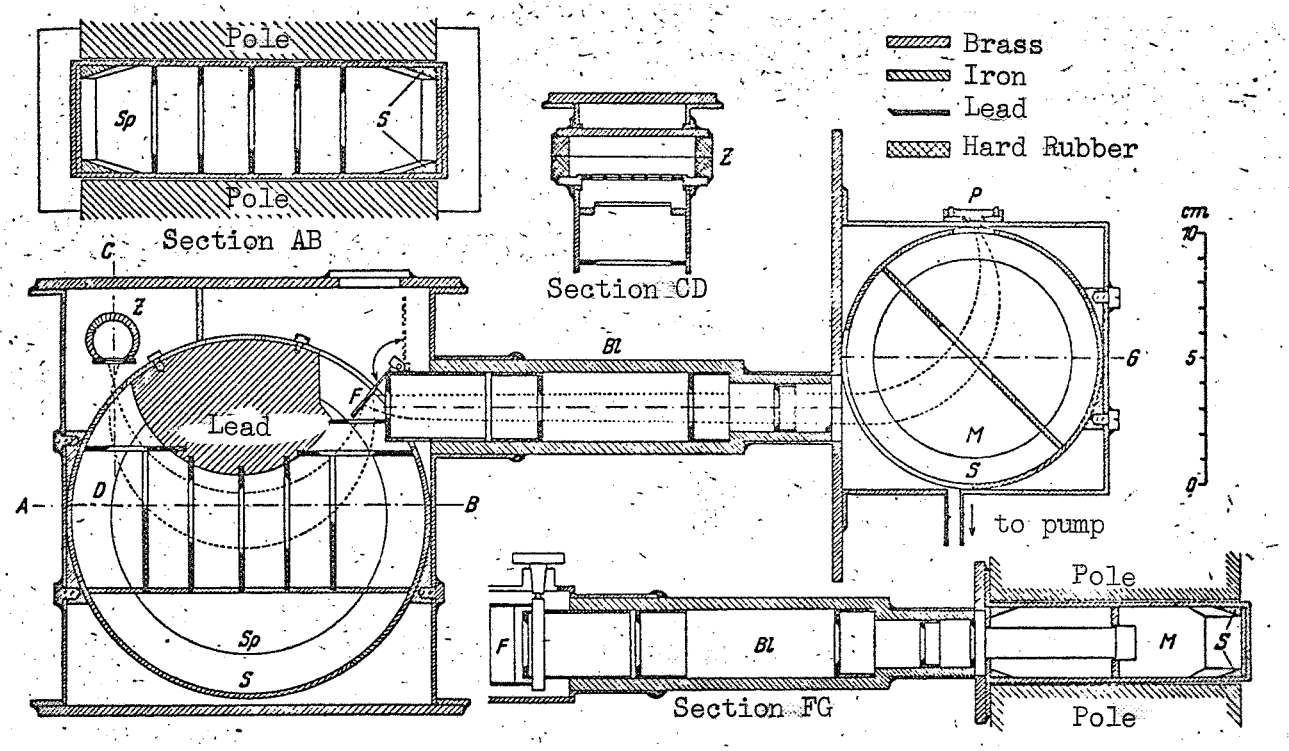

Figure 1

Experimental apparatus.

source was a powerful RaD + E preparation P (of the order of 50 mC). The RaD was deposited as the dioxide on a thin strip of platinum measuring 0.3 x 1.0 cm.[5] A reasonably homogeneous electron beam of a few 100 kev was separated from the continuous β radiation of the RaE by means of a monochromator M. This fell /89
on the scattering specimen F rotatably mounted in the half-circle spectrometer Sp. In both monochromator and spectrometer the radius of curvature was about 5 cm. Electrons scattered in the foil through at least 90° were analyzed by the spectrometer and counted by a counter tube Z. The large scattering angle was chosen to eliminate electrons scattered at electrons, since for the latter only scattering angles < 90° are possible. Since the primary electrons were deflected slightly by the magnetic field of the spectrometer even before striking the foil, both the scattering angle and the position of the scattering plane vary somewhat with variation of the magnetic field in the spectrometer. These small variations are negligilbe, however. When the spectrometer was adjusted to the primary energy, the primary electrons impinged at about 20° to the surface normal and the mean scattering angle was about 110°.

The number of particles measured was often only 10^{-8} of that leaving the preparation. Therefore, in designing the apparatus, every precaution had to be taken to ensure that all unintentional scattering and secondary radiation was excluded from the spectrometer. All the parts of the apparatus were coated with paraffin. On the free sides the spectrometer was sheathed in 5 cm of lead as γ-ray protection. The apparatus was adjusted so that when the scattering specimen was swung out the counter tube always indicated only its normal null effect. The foil was suspended freely between two thin wires, sufficiently deep in the shadow of the baffle.

The collimating monochromator was designed on the following ideal principle (Fig. 2). Imagine a point electron source at the edge and in the plane of symmetry of a circular homogeneous magnetic field of strength H and radius R, which towards the outside falls very steeply to zero. If, for example, the source lies vertically above the axis of field, simple reasoning leads to

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5. I am grateful to Dr. K. Starke for obtaining this fine preparation.

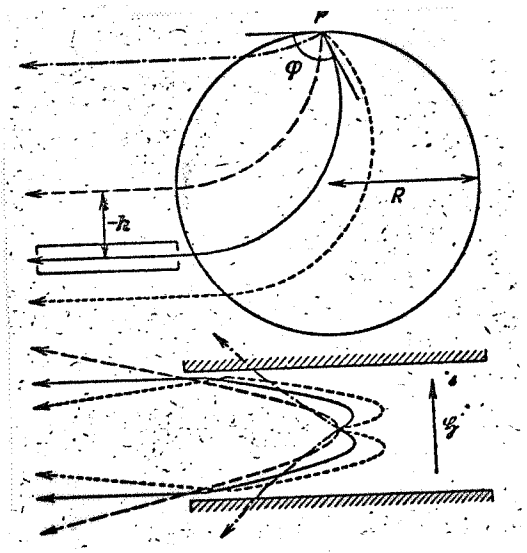

Figure 2.

Principle of collimating monochromator (schematic).

the following conclusions. /90

a. All the electrons, the radius of curvature of which is exactly equal to R, leave the magnetic field in a horizontal direction, independently of the height at which they emerge. Thus collimation is achieved in the projection on the vertical plane.

b. For any height h (reckoned from the field axis) the emerging electrons are generally still divergent or convergent in the horizontal plane. This is due to the lens action of the marginal inhomogeneity. There is, however, one

value of h, for which all the rays emerge parallel in the horizontal plane. This value is determined by the condition: $\tan \varphi = - \varphi$, where φ is the angle of entry with respect to the tangent to the edge (Fig. 2). This gives $\varphi = 116^{\circ}\ 14'$ and $h = - 0.442R$. A horizontal screening tube, applied at this level, will thus give an intense and reasonably homogeneous electron beam of rigidity HR.

This reasoning was qualitatively confirmed by vertical parallel displacement of the horizontal tube B1 (Fig. 1), after which the tube was finally fixed. Exact agreement with the calculations is not to be anticipated, for one thing because the gradient at the edge cannot be made sufficiently steep, given the necessary large pole distance. The fall from 90% to 10% of the central field occupied the equivalent of 26% of the radius. Even this could only be achieved by introducing circular iron shims S with a wedge-shaped profile into the two chambers. Furthermore, the tube B1 was made of thick iron and thus acted simultaneously as protection against the slowly diminishing coil field. The best dimensions for these parts were determined by trial and error.

The counter tube had a mica window of 2.2 mg/cm^2, the size of the aperture was 0.8 x 3.2 cm. The window recess was smoothed off on the inside with 1.5 mg/cm^2 aluminum, which is important from the point of view of the clean operation of the counter. Thus, electrons of 40 kev and below were practically unable to penetrate into the counter. The amplifier was built on the Maier-Leibnitz principle[6] and proved very successful.

The total mean resolution width of the monochromator-spectrometer system was calculated from the dimensions of the source and stops and found to be about 10% of the Hr value.

3. RESULTS

The first thing to be determined was how thin the foil

should be in order to be able to count on practically pure single scattering. Accordingly, we made measurements with three different aluminum foils at three primary energies. The spectra measured are shown in Fig. 3. Only one conversion has been made: for each primary energy all the intensities were converted in

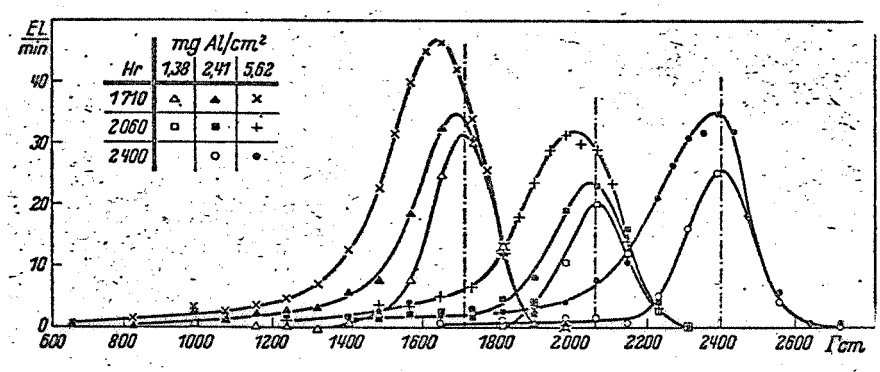

Figure 3

Scattering spectra of aluminum.

proportion to the least foil thickness used. The curves for a given primary energy should therefore coincide, if all the measurements lie in the region of single scattering. Instead of this we see that the corresponding curves are different in height and width and only the right-hand slopes coincide. The important thing, however, is the fact that, if the foil thickness is sufficiently small, for all three primary energies the distribution curve becomes symmetric about the mean primary energy, with a mean distribution width which, within the limits of accuracy of the measurements, corresponds to that calculated from the widths of the stops, namely about 10%[7]. The curves for the

7. Strict symmetry, of course, is not to be expected in view of the ray geometry.

thinnest foils also show no clearly measurable branches in the direction of lower energies, with the exception perhaps of the curve for Hr 2400 and 2.41 mg/cm^2 (see below). Thus, only these symmetric curves can be ascribed to pure single scattering. The result, therefore, is that no measurable energy losses are associated with single scattering at Al nuclei[8]. The electrons of reduced energy observed with thicker foils can thus have nothing directly to do with nuclear single scattering. /91

Table

Scattering Intensities

I'cm (kev)	Substance	mg/cm^2	N/min
1710 (210)	Al	1.38	31
	Ag	0.195	22
	Au	0.143	25
2400 (370)	Collodion	4.74	23.5
	Al	2.41	25
	Ni	1.065	30
	Ag	0.585	31
	Au	0.286	30

8. According to the theory, radiation losses are much too rare to be observed here; cf. Braunbek W. and Weinmann E.: Z. Physik, 110, 360, 1938.

Corresponding measurements were made with nickel, silver and gold; the foil thicknesses and energies are given in Table 1. In each case the thickness was chosen so that the scattering power was not too different. In all these cases only the primary line could be distinguished in the width determined by the apparatus. In no instance was it possible to detect electrons of reduced energy.

Collodion films were also investigated as representative of light-atom foils. The thinnest collodion film, with which satisfactory scattering intensity could still be obtained at an energy of 370 kev, gave the spectrum presented in Fig. 4. In contrast to equivalent foils of higher atomic number, this shows

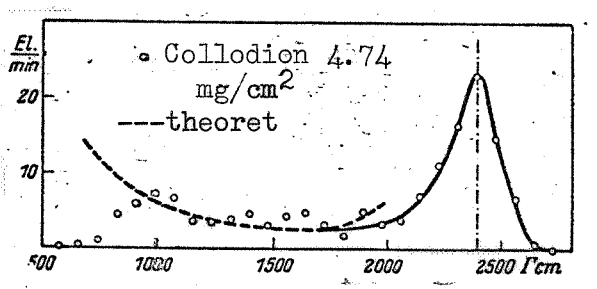

Figure 4

Scattering spectrum of a collodion film.

a slight unsymmetric broadening of the primary line, but also a distinct continuum, which at low energies appears to become more intense, especially if we take into account the increasing absorption in the counter window, as a result of which the spectrum is finally cut off completely. It would have been worth determining whether at even smaller thicknesses this branch also vanishes; unfortunately, the primary intensity was not sufficient for this. The subsequent discussion, however, will show that this vanishing is highly probable, since the continuum can easily be traced to secondary events, which have nothing directly to do with nuclear

single scattering. Analogous results were obtained with a paracyanogen $(CN)_x$ film (on a very thin film of zapon, altogether 4.0 mg/cm^2), although the principal maximum, owing to the unavoidable inhomogeneity of this layer, was broader than for collodion.

All the measurements were immediately plotted, so that the heights N of the spectral intensity maxima were also obtained with a certain accuracy. These values are grouped in Table 1. The accuracy can scarcely be better than 5%, and certainly less for the relatively inhomogeneous gold foil.

4. DISCUSSION OF ENERGY DISTRIBUTION

The main reason for the decrease in the energy of electrons in the energy range in question is interaction with the electron shells of the atoms they traverse. In general, this leads to a gradual absorption of energy by ionization and activation, but occasionally degenerates into sudden, sharp energy losses associated with large deflection angles (single scattering at electrons). The unsymmetric broadening of the primary line, observed in the spectrum of the scattered electrons, is attributable to the combined action of the gradual decrease in energy and the additional nuclear multiple scattering. On the one hand, the multiply scattered electrons may travel a very long way in the layer and thus lose a great deal of energy; on the other hand, particles that have already lost energy again undergo more large nuclear deflections. This explains the at first sight surprisingly rapid broadening of the lines with increasing layer thickness (Fig. 3), especially if we also take into account the fact that the spectrum of the ordinary energy losses in foils has a long spur in the direction of high losses[9]. /92

The effective cross section per atom for the interaction with the electron shell is roughly proportional to Z, while the effective cross section for nuclear scattering varies approximately with Z^2. Accordingly, the above-mentioned interference effects become particularly noticeable in relation to light-atom

scattering layers. Thus, it is understandable that the scattering spectrum of collodion (Fig. 4) should look different from the spectra for equivalent foils of higher atomic number. In the case of Al (2.41 mg/cm^2, Hr 2400) a continuous branch in the direction of low energies is perhaps still just indicated (Fig. 3). In the case of collodion, a semi-quantitative interpretation of this continuum can be obtained without too much trouble; it is essentially the result of single scattering at electrons. We shall make the following simplifying assumptions:

a. The cross section $d\Phi_e$ per atom for scattering at electrons can be expressed in the non-relativistic approximation as:

$$d\Phi_e = 4Z \left(\frac{e^2}{mv^2} \right)^2 \cos\vartheta \left(\frac{1}{\sin^4\vartheta} + \frac{1}{\cos^4\vartheta} - \frac{1}{\sin^2\vartheta \cos^2\vartheta} \right) 2\pi \sin\vartheta d\vartheta.$$

In accordance with elementary laws, to each scattering angle ϑ there corresponds an Hr value:

$$Hr = (Hr)_0 \cos\vartheta; \quad |d(Hr)| = (Hr)_0 \sin\vartheta d\vartheta,$$

where $(Hr)_0$ relates to the primary electrons. If we put $Hr/(Hr)_0 \equiv x$, then:

$$d\Phi_e = 8\pi Z \left(\frac{e^2}{mv^2}\right)^2 \frac{1 - 3x^2 + 3x^4}{x^3 (1 - x^2)^2} dx.$$

b. At first these electron-scattered electrons cannot enter the spectrometer, because ϑ is always less than $\pi/2$. After losing a considerable part of their energy in collisions, however, i.e., when x is plainly < 1, they suffer much stronger multiple scattering than the primary electrons. In our calculations we assume that when they finally emerge from the foil, they have an isotropic distribution of direction. The spectrum of these isotropic electrons extends over a broad range of x. The fraction entering the spectrometer $\Delta x = ax$, where a is the relative resolution width (here $a \approx 0.1$). Then for a definite Hr setting of the spectrometer, except for a constant factor, the measured number of particles

$$N_e(x) = \frac{d\Phi_e}{dx} ax \frac{\Omega}{4\pi},$$

where Ω is the apertural angle of the spectrometer.

c. For nuclear scattering we shall also confine ourselves to the Rutherford approximation. The number of measured nuclear-scattered electrons is then, except for the same constant factor as in b,

$$N_k = \frac{1}{4} Z^2 \left(\frac{e^2}{mv^2}\right)^2 \frac{\Omega}{\sin^4 \Theta/2},$$

where Θ is the mean scattering angle (here $\Theta = 110^{\circ}$).

For collodion $\bar{Z} = \Sigma Z^2 / \Sigma Z = 6.9$, so that finally:

$$\frac{N_e(x)}{N_k} = 0.052 \frac{1 - 3x^2 + 3x^4}{x^2 (1 - x^2)^2} .$$

Here N_k is the measured height of the undisturbed primary maximum, $N_e(x)$ is the ordinate of the distribution curve above x·Hr as x-axis. This function is indicated by the broken line in Fig. 4.[10] It is clear that up to about $x = 0.7$ it is compatible with the observations. The theoretical rise in the direction of lower energies is recognizable in the measurements as clearly as might be expected in view of the rapidly increasing absorption in the counter window and the low intensity.

As a basis for comparison with cloud chamber experiments we might take the measurements of Ho[1], since these lie in approximately the same energy region. According to Ho, the single-scattering of positrons in air should give a continuum extending far in the direction of low energies; the numerical data indicate that by chance this continuum must have exactly the shape of the curve X in our Fig. 3. Our measurements, however, show that in the case of pure single scattering no element gives a continuum of this shape. On the contrary, all these measurements suggest that the single-scattered electrons have exactly the primary

/93

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10. If we use the rigorous scattering formulas of Chr. Moller (Ann. Physik 14, 531, 1932) or Mott-Sexl (see section 5), the calculations show that the curve lies somewhat lower at small Hr.

energy, apart from exceptional radiation losses, which could not be detected here for intensity reasons. Even the results for collodion (Fig. 4) can hardly be given the interpretation in question, since, on the one hand, the shape of the continuum is quite different, and, on the other, as indicated above, this continuum is most likely attributable to secondary events, which ought to disappear on using even thinner foils. Nevertheless, more accurate experiments with light elements would not be inappropriate.

None of the cloud chamber experiments was carried out under exactly the same conditions as the present spectrometric measurements. Nevertheless, the results of the cloud chamber experiments must now be judged rather more critically. Our measurements show how easily sudden large energy losses can be simulated by a superimposed gradual decrease in velocity. It would appear to be not impossible that so far this influence has been underestimated in measuring cloud chamber tracks. If we consider, say, the conditions under which the curve X of Fig. 3 was recorded, then, according to White and Millington[9], if the foil were traversed at right angles, the most frequent Hr loss should be not more than 50 Γ cm. As can be seen, the actual losses are distributed over a much greater range. Since the decrease in energy is almost proportional to the mass, this foil is only equivalent to 4.6 cm of air. The distances used to measure a cloud chamber track on either side of a deflection point will often be even greater. Nevertheless, it would be desirable to supplement the present experiments with corresponding experiments using high-energy electrons and positrons.

5. DISCUSSION OF SCATTERING INTENSITIES

According to Dirac's theory of the electron, the classical Rutherford formula for the single scattering of electrons must be supplemented by a factor f, designed to take into account the spin and relativity. Mott[2] has obtained the following expression for this factor:

$$f_M = 1 - \beta^2 \sin^2 \frac{\Theta}{2} + \frac{\pi\beta Z}{137} \sin \frac{\Theta}{2} \cos^2 \frac{\Theta}{2},$$

where Θ is the scattering angle. Following different paths, Sexl[3] and Urban[4] arrived at the following slightly modified formula:

$$f_S = 1 - \beta^2 \sin^2 \frac{\Theta}{2} + \frac{\pi\beta Z}{137} \sin \frac{\Theta}{2}.$$

Both formulas are to be understood as the beginnings of series expansions in powers of Z/137; higher terms have not yet been calculated. The straight lines in Fig. 5 represent the function f(Z) as given by both these formulas, for our scattering angle of 110° and for two different primary energies. We have plotted relative values of f referred to Al taken as 1.

These relative values can also be derived from the measurements presented in Table 1. In this connection we can freely assume that pure single scattering was the rule. This is indicated by the experiments with foil of variable thickness, but Wentzel's criterion of single scattering also appears to be well satisfied, even if we reckon with double the thickness of foil. Even an estimate based on a formula presented by Chase and Cox[11], which gives the spread of the angular distribution due to the superimposed multiple scattering, shows that in our case this effect cannot exceed the limits of error of the measurements. Finally, by way of precaution, the various foils were selected so that, as shown in Table 1, their scattering power was much the same. Accordingly even if a small component due to multiple scattering was also measured, it would in all cases be of about the same magnitude and would subsequently cancel out. /94

According to the Rutherford formula, the scattering cross section should be proportional to Z^2. We therefore form expressions:

$$F = \frac{N}{\sigma} \frac{\Sigma A}{\Sigma Z^2},$$

where N is the measured scattering intensity in accordance with Table 1, σ is the density by surface, A the atomic weight, and Z the atomic number of the atoms in the foil. Then:

$$F/F_{Al} = f/f_{Al}.$$

These experimental relative values of f are presented in Fig. 5

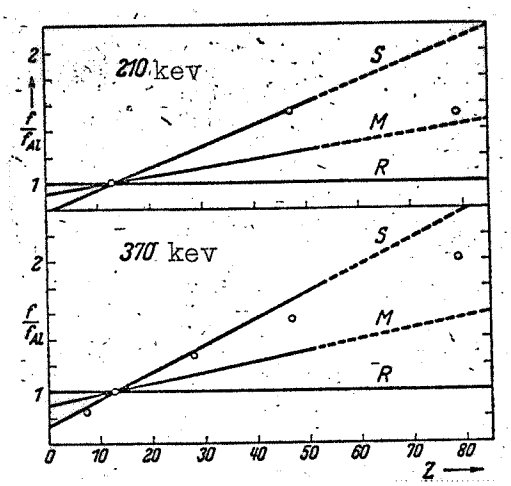

Figure 5

f = correction factor for Rutherford formula = const x scattering cross section/Z^2. Theoretical curves: S after Sexl-Urban, M after Mott, R after Rutherford.

Accordingly, the value for collodion is plotted for $\bar{Z} = \Sigma Z^3/\Sigma Z^2 =$ $= 7.3$. It is immediately obvious that, in general, when compared with that for aluminum, the experimental scattering cross section depends more heavily on the atomic number than might be expected from the Rutherford formula; this is required by the theory. For a more exact comparison with the two theoretical formulas the points for Z = 79 (Au) must be excluded, since the theoretical formulas are not valid for such high atomic numbers. We then see that Sexl's formual reproduces the experimental results much better than the original formula proposed by Mott*.

In principle, we can also use the energy dependence of the scattering cross section to decide between the two formulas. Unfortunately, the measuring accuracy we were able to achieve was not good enough for this purpose. From a not too accurate measurement of the primary intensities coming from the monochromator, we were able to conclude that the energy dependence is roughly consistent with both formulas.

In general, therefore, the experiments reported indicate that in the energy range of a few 100 kev the Dirac theory does justice to the phenomena of nuclear scattering. As for nitrogen, about which there has been so much discussion, our measurements on collodion and paracyanogen show that, at least in the region of a few 100 kev, it is in no way an exception, since in these measurements nitrogen's contribution to the scattering power was theoretically about 12% for collodion and more than 50% for paracyanogen.

Among the numerous earlier investigations of the single scattering of electrons the following are of particular interest.

Neher[12] has measured absolute values in the energy range from

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* Note added in proof: In the meantime this question has been settled by the extension of Mott*s formula to the fourth order in Z/137 (McKinley W. A. and Feshbach H.: Physic. Rev. 74, 1759, 1948). The extended formula is compatible with our results, with the exception of those for gold, for which the theoretical points in Fig. 5 lie at 2.74 (210 kev) and 2.90 (370 kev). Although even the fourth approximation is still poor, and the accuracy of the measurements may only be 10%, this considerable deviation is still remarkable.

50 to 150 kev at angles of scattering > 90°. As Urban[4] has shown, these results fit Sexl's formula more closely than that of Mott, even though the agreement with the former is not altogether perfect. Van de Graaff and coworkers[13] have made some very extensive absolute measurements in the range 1.5-2.3 Mev which fully confirm Mott's formula; the angle of scattering was too small to decide between the Mott and the Sexl formulas, however. In an investigation interrupted by the war (1-2.4 Mev, small scattering angles), Bothe and Ratzel[14] found deviations from the theory quite similar to those which Van de Graaff and coworkers first postulated, but were later unable to confirm. These experiments are now being repeated and continued.

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* See footnote in text.

6. Maier-Leibnitz H.: Rev. Sci. Instruments, 19, 500, 1948 (Mr. Maier-Leibnitz was kind enough to lend me this amplifier).
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* See footnote in text.